

Functional hand proportion is approximated by the Fibonacci series

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The debatable relationship of functional human hand proportion with the Fibonacci series has remained an obscure scientific enigma short of clinical interest. The main difficulty of proving such a relationship lies in defining what should constitute true “functional” proportion. In this study, we re-evaluate this unique relationship using hand flexion creases as anatomical surrogates for the functional axes of joint rotation. Standardised desktop photocopies of palmar views of both hands in full digital extension and abduction were obtained from 100 healthy male volunteers of Chinese ethnicity. The functional axes were represented by the distal digital crease (distal interphalangeal joint, DIPJ), proximal digital crease (proximal interphalangeal joint, PIPJ), as well as the mid-point between the palmar digital and transverse palmar creases (metacarpophalangeal joint, MCPJ). The ratio of DIPJ-Fingertip:PIPJ-DIPJ:MCPJ-PIPJ (p3:p2:p1) was measured by two independent observers and represented as standard deviation about the mean, and then compared to the theoretical ratio of 1:1:2. Our results showed that, for the 2nd to 5th digits, the p2:p3 ratios were 0.97 ± 0.09 , 1.10 ± 0.10 , 1.04 ± 0.12 , and 0.80 ± 0.08 , respectively; whilst the p1:p2 ratios were 1.91 ± 0.17 , 1.98 ± 0.14 , 1.89 ± 0.16 , and 2.09 ± 0.24 , respectively. When the data were analysed for all digits, they showed a combined p3:p2:p1 ratio of 1:0.98:2.01. In conclusion, our results suggest that functional human hand proportion, as defined by flexion creases, is approximated by the Fibonacci series. (Folia Morphol 2012; 71, 3: 148–153)

Key words: 2D:4D ratio, littler series, golden proportion

INTRODUCTION

The study of human hand proportion is rapidly gaining a standing in modern medicine as a result of strong evidence linking naturally occurring variations with diseases. Recent investigations have demonstrated that the index to ring finger length (2D:4D) ratio may be associated with several human diseases. The 2D:4D ratio — a sexually-dimorphic trait diminished in males compared to females — was al-

ready well-established a century ago [2]. However, it was not only recently that its physiological basis was attributed to a prenatal origin, probably as a result of high androgen exposure [35]. The 2D:4D ratio also demonstrates lateral asymmetry, and displays greater sex difference in the right hand [13]. Candidate genes such as *HOX* [16] and *LIN28B* [23], as well as CAG trinucleotide repeat polymorphisms of the androgen receptor *AR* gene [21], have been

proposed as responsible for these observations. Most interestingly, the 2D:4D ratio has recently been associated with a wide spectrum of diseases and psychological traits. Several authors have reported that a lower “masculinised” 2D:4D ratio is linked to attention-deficit-hyperactivity disorder [29], alcohol dependency [17], autism [19], osteoarthritis [34], myocardial infarction [20], prostate cancer [27], and congenital adrenal hyperplasia [25]. In contrast, a higher “feminised” 2D:4D ratio has been implicated with greater risk for oral cancer [24] as well as androgen insensitivity syndrome [5].

The “Fibonacci proportion” of the human hand phalanges, however, has not been as widely researched, and its existence remains highly controversial. The Fibonacci series was originally described by Leonardo Pisano Bigollo as a sequence of numbers generated by the sum of the two preceding numbers, i.e. 0, 1, 1, 2, 3, 5, 8 and so forth [30]. As the series progresses, the ratio of each term to the previous term approaches 1.618 as a limit, a value popularised as the Golden mean due to its association with ubiquitous structures in nature such as the spirals of the galaxies, seashells, flowers, and DNA structure. Most intriguingly, the ratio has been implicated in the generation of various proportions of the human anatomy including the hand, heart, and face, which may be related to its role in form and functional optimisation [1, 7–9, 18].

It was initially observed by Thompson that naturally occurring logarithmic curves occur in the path of motion of digits [32]. Subsequently, eminent hand surgeon Dr. J. William Littler inferred from this observation that the functional motion path of human fingertips would approximate an equiangular spiral with a generating radius of 1.618, and that the bone lengths would follow the Fibonacci relationship [18]. Remarkably, the arciform motion path of unrestrained digit flexion and extension had been confirmed to follow an equiangular spiral using motion analysis systems [11]. In contrast, results from two earlier studies attempting to associate inter-articular bone lengths or joint rotation axes with the Fibonacci series failed to demonstrate the latter relationship proposed by Littler [12, 26]. The major problem in addressing the hypothesis lay in the interpretation of what constitutes the true “functional” proportion of the human hand, and the consequent methodology of measuring it using a suitable surrogate. True functional proportion should refer to a spatial organisation that is closely related to the natural and unrestrained motion paths of the

hand [11]. Furthermore, previous studies had incorrectly assumed that the ratios would approximate the Fibonacci series originating from 2, 3, 5, and so forth, when the original hypothesis had made no such assumption [26]. Finally, the spatial organisation of the hand has been shown to vary according to factors such as ethnicity, age, gender, hand preference, lateralisation, and *in utero* hormone exposure [10], suggesting that deviations from empirical models would be expected.

Although it was suggested that the functional lengths of the phalanges, determined by the distance between centres of rotation of the joints, should yield values that follow the Fibonacci series [26], what constitutes the best surrogate for the centre of rotation of the respective joints remains unclear. In this paper, we re-evaluate the mathematical validity of the Fibonacci relationship with respect to the functional lengths of human hand phalanges, using the palmar and digital flexion creases of the hand as anatomical surrogates for the functional axes of rotation of the hand joints. Based on the above assumptions, we found a pattern that perhaps support the hypothesis of Littler — that the length ratio of the distal, middle and proximal human hand phalanges of each digit follows the Fibonacci sequence of 1:1:2.

MATERIAL AND METHODS

Participants

Hundred healthy male volunteers were recruited, and appropriate consent was obtained from all participants. Exclusion criteria included any prior injury, trauma, or surgery to the hand resulting in deformity, or any obvious congenital deformities of the hand. Participants remained anonymous, and no clinical information was obtained apart from their age. The mean age of these participants was 21 (range 17–30) years.

Outcome measures

Using a desktop photocopier, palmar views of the entire human hand in full digital extension and abduction were obtained directly by placing both hands faced down onto the scanner. Where necessary, participants were asked to remove jewellery that would compromise measurement. The functional joint axes were represented by the distal digital crease (distal interphalangeal joint, DIPJ), proximal digital crease (proximal interphalangeal joint, PIPJ), as well as the midpoint between the palmar

Table 1. Intra- and inter-observer variation (Bland-Altman statistic)

		Mean [mm]	SD	95% CI
Inter-observer		-0.02	0.08	-0.08 to 0.04
Intra-observer	Observer 1	0	0.12	-0.08 to 0.08
	Observer 2	0.02	0.12	-0.07 to 0.10

digital and transverse palmar creases (metacarpophalangeal joint, MCPJ) (Fig. 1). The distances between the MCPJ and PIPJ (MCPJ-PIPJ), the PIPJ and DIPJ (PIPJ-DIPJ), as well as the DIPJ and fingertip (DIPJ-Tip) were measured for each finger ray. The ratios of DIPJ-Tip:PIPJ-DIPJ:MCPJ-PIPJ (p3:p2:p1) were compared to the theoretical ratio of 1:1:2. The lengths of digits 2D (index finger) and 4D (ring finger) were also measured from the most proximal crease of each digit to the fingertip. 2D:4D ratio was obtained by simply dividing 2D length by 4D length.

Statistical analyses

The measurements were obtained independently by two examiners (C.J.Y. and C.K.W.) using a metered ruler, and approximated to the nearest half-millimetre. Intra- and inter-observer variation was estimated via the Bland-Altman method [6], using

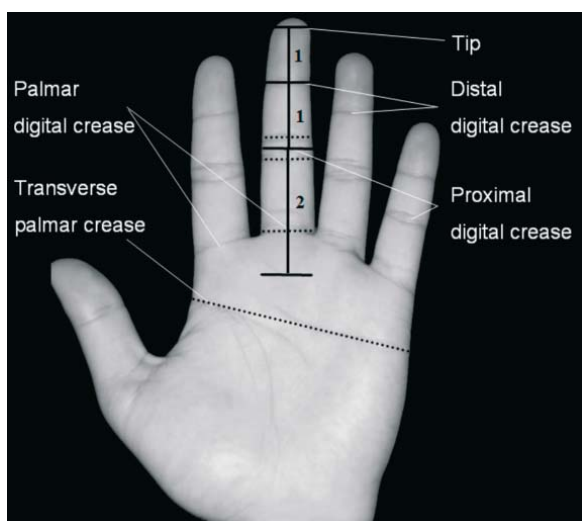


Figure 1. The functional joint axes were represented by the distal digital crease (distal interphalangeal joint, DIPJ), proximal digital crease (proximal interphalangeal joint, PIPJ), as well as the midpoint between the palmar digital and transverse palmar creases (metacarpophalangeal joint, MCPJ). The ratio of DIPJ-Tip:PIPJ-DIPJ:MCPJ-PIPJ was compared to the theoretical ratio of 1:1:2 in accordance with the Fibonacci series (not drawn to scale).

duplicate measurements from 10 randomly selected index fingers. For each observer, the mean, standard deviation, and 95% confidence intervals were reported. The ratios of DIPJ-Tip:PIPJ-DIPJ:MCPJ-PIPJ were calculated and represented as standard deviation about the means. Paired student's t-tests were used for right-left comparisons. Pearson's correlation coefficient was used to identify any correlation between the digit ratios. A two-tailed $p < 0.05$ was taken to be statistically significant.

RESULTS

The results of the intra- and inter-observer variation are shown in Table 1. They suggest that both observers reliably report the same distance when it is measured on different occasions. There is good agreement between the authors when measuring the same distance on the same hand images.

The summary of the results is shown in Tables 2 and 3. For the 2nd to 5th digits, the p2:p3 ratios were 0.97 ± 0.09 , 1.10 ± 0.10 , 1.04 ± 0.12 , and 0.80 ± 0.08 , respectively; whilst the p1:p2 ratios were 1.91 ± 0.17 , 1.98 ± 0.14 , 1.89 ± 0.16 , and 2.09 ± 0.24 , respectively. Lateral asymmetry was observed for the 3rd digit p2:p3 (right = 1.09 ± 0.10 , left = 1.11 ± 0.11 ; $p = 0.0004$) and p1:p2 ratios (right = 1.99 ± 0.14 , left = 1.96 ± 0.14 ; $p = 0.0038$), as well as the 5th digit p2:p3 ratio (right

Table 2. Average ratios of distances as approximated by flexion creases of the human hand

Digits	PIPJ-DIPJ/DIPJ-Tip	MCPJ-PIPJ/PIPJ-DIPJ
Index	0.97 ± 0.09	1.91 ± 0.17
Middle	1.10 ± 0.10	1.98 ± 0.14
Ring	1.04 ± 0.12	1.89 ± 0.16
Little	0.80 ± 0.08	2.09 ± 0.24
Combined	0.98 ± 0.15	1.97 ± 0.20

DIPJ — distal interphalangeal joint; MCPJ — metacarpophalangeal joint; PIPJ — proximal interphalangeal joint

Table 3. Lateral asymmetry in digit ratios of the human hand

Digits	PIPJ-DIPJ/DIPJ-Tip			MCPJ-PIPJ/PIPJ-DIPJ		
	Left	Right	P	Left	Right	P
Index	0.9746 ± 0.09128	0.9724 ± 0.09298	0.7483	1.9041 ± 0.1652	1.9223 ± 0.1694	0.1786
Middle	1.1135 ± 0.1064	1.0870 ± 0.1011	0.0004	1.9585 ± 0.1385	1.9925 ± 0.1412	0.0038
Ring	1.0374 ± 0.1038	1.0363 ± 0.1303	0.9275	1.8876 ± 0.1485	1.8986 ± 0.1749	0.3811
Little	0.8203 ± 0.08251	0.7886 ± 0.08414	< 0.0001	2.0957 ± 0.2381	2.0839 ± 0.2457	0.6087
Combined	0.9864 ± 0.07477	0.9711 ± 0.08198	0.0011	1.9615 ± 0.1291	1.9743 ± 0.1508	0.1555

DIPJ — distal interphalangeal joint; MCPJ — metacarpophalangeal joint; PIPJ — proximal interphalangeal joint

= 0.79 ± 0.08, left = 0.82 ± 0.08; $p < 0.0001$). When the data were analysed together for all digits, they showed an approximate p3:p2:p1 ratio of 1:0.98:2.01. Lateral asymmetry was demonstrated for the combined p2:p3 ratio (right = 0.97 ± 0.08, left = 0.99 ± 0.07; $p = 0.0011$) but not for the p1:p2 ratio (right = 1.97 ± 0.15, left = 1.96 ± 0.13; $p = 0.16$). We tested for any association between the Fibonacci phalangeal length ratios and the 2D:4D ratio. However, we did not observe any linear correlation between the 2D:4D ratio and any of the Fibonacci ratios (p1:p2 or p2:p3) for all digits of right and left hands ($p > 0.05$ for all Pearson's coefficients).

DISCUSSION

Our study attempted to prove Littler's hypothesis that the length ratio of the distal, middle and proximal human hand phalanges of each digit follows the Fibonacci sequence of 1:1:2. Through our investigation, we demonstrated that the human hand digit proportion does approximate the Fibonacci ratio of 1:1:2, and perhaps ascertained Littler's hypothesis. Littler, in his classic article "On the adaptability of man's hand", illustrated how the functional flexion-extension motion of the fingertips progressively traces an equiangular spiral in space that has a curvature related to the natural form of the Fibonacci series. However, the anthropomorphic structure of the human hand has been criticised to be incongruent with the ability to generate this spiral mathematically. Indeed, Park et al. [26] concurred that bone lengths of fingers do not follow the Fibonacci series but suggested that if "functional" lengths of the joints, as determined by their centres of rotation, are considered instead, they yield lengths that do in fact mathematically follow the Fibonacci relationship. Independently, Hamilton and Dunsmuir [12] also failed to

prove the relationship by examining functional lengths corresponding to distances between the transverse axes of joint rotation, which were derived from anatomical hand specimens dissected to leave only the bones, ligaments, and joint capsules. It may be that study designs based solely on skeletal anatomy without taking into consideration surrounding musculature, soft tissue, and skin were inadequate since the dynamic action of these structures would be expected to affect the joint motion axes significantly. Furthermore, the centres of rotation change during flexion-extension [4, 31]. Thus, we propose that a surrogate "functional" centre of rotation must account for these composite factors.

In selecting a suitable surrogate for the functional axes of rotation of the hand joints, it is prudent that the surrogate be able to describe the spatial organisation that is rooted to its functional adaptability, and be representative of its form and function as would be optimised by the Fibonacci series. Notably, palmar and digital flexion creases represent periarticular tissue strain from tensile and compressive forces during joint movement, formed by the subcutaneous tissue folds in the hands, which are connected to the underlying articulations. Previous studies obtaining the joints' centres of rotation solely from examining bone alone would therefore not be entirely representative, since the soft tissue and epidermal components constitute an important aspect of hand form and function. Formation of flexion creases occurs between the 8th and 13th gestational weeks in close morphologic relationship to the foetal volar pads, and both primary genetic determinants and development secondary to flexion function have been proposed as the mechanisms underlying crease development [3]. The latter was also supported by studies that demonstrated close correspondence between the appearance of flexion

creases with hand or digital movement [15, 28]. These data suggest that palmar and digital flexion creases are appropriate surrogates for the functional axes of rotation of the hand joints.

Interestingly, a subsequent re-interpretation of the results of Hamilton and Dunsmuir [12] showed that the ratio of $p_3:p_2:p_1$ approximated 1:1:2 for the little finger, and 1:1.3:2.3 for the other fingers [14]. Such a relationship is characteristic of a general class of summative sequences known as Lucas sequences, in which the ratio between a term and the preceding one approaches 1.618 as a limit (the Fibonacci sequence is one example). This may yet imply that functional lengths of the hand — both skeletal and whole, simply follow variations of Lucas sequences in their spatial organisation, perhaps as an optimisation algorithm for form and function development [9]. The exact implications on hand biomechanics and treatment of injuries, however, remain to be investigated. Severe traumatic injury to the hand, such as shortening of the finger from fractures, may result in an altered proportion, which in turn may be associated with functional impairment. It would be interesting to determine if large deviation from Fibonacci proportion post-injury or surgical repair is related to poor functional outcome. However, although we recognise that the Fibonacci sequence may be the guiding principle behind anatomical hand development, the individual may have already developed learned compensatory or adaptive mechanisms to cope with any congenital deviation. Thus, functional restoration may be better achieved with repair to the original state rather than to a mathematically perfect Fibonacci proportion. In the absence of such knowledge, however, the surgeon may then be guided in principle.

There is increasing evidence supporting the utility of hand proportion in the understanding of pathophysiology, as well as in diagnoses of certain human diseases. Notably, the 2D:4D ratio has been associated with a wide spectrum of diseases and psychological conditions. The 2D:4D ratio is believed to be genetically-linked, by virtue of its familiar resemblance [33] and ethnic variation [22]. Like the 2D:4D ratio, hand flexion creases, and consequently the Fibonacci proportions, are probably genetically-determined as well [3]. We did not observe any correlation between the 2D:4D ratio with any of the Fibonacci ratios ($p_1:p_2$ or $p_2:p_3$) for all digits of right and left hands. This may imply that the genetic or developmental bases of both ratios are independent of each other. Going further, given the association

of the Fibonacci proportion with several important regions of human anatomy, such as the face [1], myocardium [7], and coronary arteries [9], there may exist an underlying empirical scheme and physico-mathematical law governing the optimal form and function development of these structures [8]. The implications of the Fibonacci proportion may go beyond local biomechanics of the human hand, and any local deviation from the ideal proportion may in turn be a manifestation of generalised spatial disorganisation and global systemic dysfunction. Nonetheless, our study is limited by the relatively small cohort of young males of Chinese ethnicity. Since hand proportion has been known to vary according to age group, gender, and ethnicity [10], the conclusions drawn from our study need to be interpreted with caution.

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